Physics III
ISI B.Math
Final Exam : November 23, 2009

Total Marks: 100
Answer any five questions.

1. (i) Find the potential corresponding to the electric field given below. $\mathbf{E} = k[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}]$ (5)
(ii) A point charge is at a distance d from a grounded infinite conducting plane. How much energy is required to move the charge infinitely far from the plane ? (5)
(iii) Consider two concentric spherical metal shells of radii r_1 and r_2. If the outer shell has a charge q and the inner shell is grounded, what is the charge on the inner shell ? (5)
(iv) A uniform electric field E_0 in the x direction is produced by an appropriate charge configuration. A thin sheet of charge σ per unit area is placed perpendicular to the x- direction at $x = 0$. If the initial charge configuration is assumed to be undisturbed by the presence of the sheet, what is the total electric field on either side of the sheet? (5)

2. (a) A conductor at potential $V = 0$ has the shape of an infinite plane except for a hemispherical bulge of radius a. A charge q is placed at a distance p from the plane (or a distance $p - a$ from the top of the bulge)(Fig 2a). What is the force on the charge ?(5)
(b) Two spherical cavities , of radius a and b are hollowed out from the interior of a (neutral) conducting sphere of radius R. At the center of each cavity a point charge is placed, call these charges q_a and q_b. (Fig 2b)(15)
(i) Find the surface charges σ_a, σ_b and σ_R.

(ii) What is the electric field outside the conductor?
(iii) What is the field within each cavity?
(iv) What is the force on q_a and q_b ?
(v) Which of these answers would change if a third charge q_c were brought near the conductor ?

3. (a) In 1897 J.J. Thomson “discovered” the electron by measuring the charge-to-mass ratio of cathode rays as follows
(i) First he passed the beam through uniform crossed electric and magnetic fields **E** and **B**(mutually perpendicular and both of them perpendicular to the beam) and adjusted the electric field until he got zero deflection. What, then, was the speed of the particles (in terms of E and B)?(5)
(ii) Then he turned off the electric field, and measured the radius of curvature R, of the beam, as deflected by the magnetic field alone. In terms of E, B and R, what is the charge-to-mass ratio ($\frac{q}{m}$) of the particles ?(5)
(b) Three charges are situated at the corners of a square of side a.
(i) How much work does it take to bring in another charge $+q$ from far away and place it in the fourth corner?(5)
(ii)How much work does it take to assemble the whole configuration of charges ? (5)

4. a) Write down the full set of Maxwell's equations in differential form.(4)
b) Explain what inconsistency led Maxwell to add the ”displacement current” term to modify Amperes Law and how consistency is restored on addition of this term. (6)
c) Show that a magnetic field **B** and electric field **E** that is a solution to Maxwell's equations can always be written as (5)

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

where ϕ is a scalar function and **A** is a vector field.
(d) Show that , for Maxwell's equations in vacuum, each Cartesian component of **E** and **B** satisfies the 3-D wave equation (5)

$$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

with $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

5. A very small circular loop of radius a is initially coplanar and concentric with a much larger loop of radius b ($a \ll b$). A constant current I is passed in the large loop, which is kept fixed in space, and the small loop is rotated with angular velocity ω about a diameter. The resistance of the small loop is R and its self-inductance is negligible.
(a) Calculate the current in the small loop as a function of time. (6)
(b) Calculate how much torque must be exerted on the small loop in order to rotate it.(7)
(c) Calculate the induced emf in the large loop as a function of time.(7)

6. (a) Consider the following situation. A line charge λ is glued onto the rim of a wheel of radius b , which is then suspended horizontally, as shown in the figure, so that it is free to rotate. (The spokes are made of some non-conducting material). In the central region, out to the radius a, there is a uniform magnetic field $\mathbf{B_0}$ pointing up. Now someone turns the magnetic field off. It is observed that the wheel starts to rotate. Explain why this happens, find the direction of rotation and the angular momentum acquired by the wheel. It appears that the system acquired an angular momentum without the application of an external torque. Explain whether this is consistent with the conservation of angular momentum. (7)
(b) Write down the real electric and magnetic fields for a monochromatic plane wave of amplitude E_0 , frequency ω and phase angle zero that is traveling in the negative x direction and polarized in the z-direction. Find the time average (over a cycle) of the energy density and the Poynting vector for such a wave. What does the Poynting vector represent physically ? (13)